Alpha, Beta, and Now…Gamma

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Abstract

When it comes to generating retirement income, investors arguably spend the most time and effort on selecting "good" investment funds/managers—the so called alpha decision—as well as the asset allocation, or beta, decision. However, alpha and beta are just two elements of a myriad of important financial planning decisions for the average investor, many of which can have a far more significant impact on retirement income.

We present a concept that we call "Gamma" designed to quantify the additional value that can be achieved by an individual investor from making more intelligent financial planning decisions. We measure value through a certainty-equivalent utility-adjusted retirement income metric. Gamma will vary for different types of investors and for different strategies; however in this paper we focus on five fundamental financial planning decisions/techniques: a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products (i.e., annuities), tax-efficient decisions, and liability-relative asset allocation optimization.

Using Monte Carlo simulation, we estimate a retiree can expect to generate 22.6% more in certainty-equivalent income using a Gamma-efficient retirement income strategy when compared to our base scenario, which assumes a 4% initial portfolio withdrawal where the withdrawal amount is subsequently increased by inflation and a 20% equity allocation portfolio. This addition in certainty-equivalent income has the same impact on expected utility as an annual arithmetic return increase of +1.59% (i.e., Gamma equivalent alpha), which represents a significant improvement in portfolio efficiency for a retiree. Unlike traditional alpha, which can be hard to predict and is a zero-sum game, we find that Gamma (and Gamma equivalent alpha) can be achieved by anyone following an efficient financial planning strategy.

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The potential benefits from “good” financial planning decisions are often difficult to quantify. For any given portfolio, investment decisions can generally be decomposed into two primary components: beta and alpha. Beta can generally be defined as the systematic risk exposures of the portfolio (usually achieved through asset allocation), while alpha is the residual, or skill/luck-based, component associated with the various flavors of active management (e.g. tactical asset allocation, security selection, etc.). Alpha and beta are at the heart of traditional performance analysis; however, as we show in this paper, they are only one of the many important financial planning decisions, such as savings and withdrawal strategies, that can have a substantial impact on the retirement outcome for an investor.

In this paper we present a concept that we call “Gamma” designed to measure the value added achieved by an individual investor from making more intelligent financial planning decisions. This value is measured by calculating the certainty-equivalent utility-adjusted retirement income across different scenarios. The Greek letter gamma is a widely used parameter in certain areas of finance, such as derivative’s trading and risk management. We are using it here because we hope that our proposed measure will become well known, like alpha and beta; however with a specific new meaning within the context of wealth management.

We focus on five important financial planning decisions/techniques: a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products (i.e., annuities), tax-efficient allocation decisions, and a portfolio optimization that includes a proxy for the investor’s implicit and/or explicit liabilities. Each of these five Gamma components creates value for retirees, and when combined, can be expected, given the paper’s assumptions about risk aversion and other variables, to generate 22.6% more certainty-equivalent income when compared to a simplistic static withdrawal strategy according to our analysis. This additional certainty-equivalent income has the same impact on expected utility as an arithmetic “alpha” of 1.59% (i.e., Gamma equivalent alpha) and thereby represents a significant potential increase in portfolio efficiency (and retirement income) for retirees.

Beyond Beta and Alpha

The notions of beta and alpha (in particular alpha) have long fascinated financial advisors and their clients. Alpha allows a financial advisor to demonstrate (and potentially quantify) the excess returns generated, which can help justify fees. In contrast, beta (systematic risk exposures) helps explain the risk factors of a portfolio to the market, i.e., the asset allocation.

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If an advisor is paid solely to manage a portfolio of assets, and does nothing else, i.e., offers no additional advice regarding anything other than the investment of the client assets, then the concepts of alpha and beta should be relatively good measures of the value of the advisor. However, in more complex engagements, in when providing financial planning services to clients, value cannot be defined in such simple returns as alpha and beta, since the objective of an individual investor is typically to achieve a goal, and that goal is most likely to achieve a successful retirement.

It may be that a financial advisor generates significant negative alpha for a client (i.e., invests the client's money in very expensive mutual funds that underperform), but still provides other valuable services that enable a client to achieve his or her goals. While this financial advisor may have failed from a pure alpha perspective, the underlying goal was accomplished. This is akin to losing a battle but winning the war.

Individual investors invest to achieve goals (typically an inflation-adjusted standard of living), and therefore doing the things that help an investor achieve those goals (i.e., adding Gamma) is a different type of value than can be attributed to alpha or beta alone, and is in many ways more valuable. Therefore, asset-only metrics are an incomplete means of retirement strategy performance.

Gamma Factors
In this paper, we examine the potential value, or Gamma, that can be obtained from making five different "intelligent" financial planning decisions during retirement. A retiree faces a number of risks during retirement, some of which are unique to retirement planning and are not concerns during accumulation. These five different Gamma factors are:

1. *Asset Location and Withdrawal Sourcing:*
Tax-efficient investing for a retiree can be thought of in terms of both "asset location" and intelligent withdrawal sequencing from accounts that differ in tax status. Asset location is typically defined as placing (or locating) assets in the most tax-efficient account type. For example, it generally makes sense to place less tax-efficient assets (i.e. the majority of total return comes from coupons/dividends taxed as ordinary income), such as bonds, in retirement accounts (e.g., IRAs or 401(k)s) and more tax-efficient assets (i.e. the majority of total return comes from capital gains taxed a rate less than ordinary income), such as stocks, in taxable accounts. When thinking about withdrawal sequencing, it typically makes sense to withdrawal monies from taxable accounts first and more tax-efficient accounts (e.g., IRAs or 401(k)s) later.

2. *Total Wealth Asset Allocation:*
Most techniques used to determine the asset allocation for a client are relatively subjective and focus primarily on risk preference (i.e., an investor's aversion to risk) and ignore risk capacity (i.e., an investor's ability to assume risk). In practice, however, we believe asset allocation should be based on a combination of risk preference and risk capacity, although primarily risk capacity. We determine an investor's risk capacity by evaluating his or her total wealth, which is a combination of human capital (an investor's future potential savings) and financial capital. We can then either use the market portfolio as the target aggregate asset allocation for each investor (as suggested by the Capital Asset Pricing Model) or build an investor-specific asset allocation that incorporates an investor's risk preferences. In both approaches, the financial assets are invested, subject to certain constraints, in order to achieve an optimal asset allocation that takes both human and financial capital into account.

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3. *Annuity Allocation:*
Outliving one's savings is one of the greatest fears among retirees. For example, a study by Allianz Life has noted that more retirees feared outliving their resources (61%) versus death (39%) (Bhojwani 2011). Annuities allow a retiree to hedge away longevity risk and can therefore improve the overall efficiency of a retiree's portfolio. The contribution of an annuity within a total portfolio framework, (benefit, risk, and cost) must be considered before determining the appropriate amount and annuity type.

4. *Dynamic Withdrawal Strategy:*
The majority of retirement research has focused on static withdrawal strategies where the annual withdrawal during retirement is based on the initial account balance at retirement, increased annually for inflation. For example, a "4% Withdrawal Rate" would really mean a retiree can take a 4% withdrawal of the initial portfolio value and continue withdrawing that amount each year, adjusted for inflation. If the initial portfolio value was $1 million, and the withdrawal rate was 4%, the retiree would be expected to generate $40,000 in the first year. If inflation during the first year was 3%, the actual cash flow amount in year two (in nominal terms) would be $41,200. Under this approach the withdraw amount is based entirely on the initial income target, and is not updated based on market performance or expected investor longevity. The approach we use in this paper, originally introduced by Blanchett, Kowara, and Chen (2012), determines the annual withdrawal amount annually based on the ongoing likelihood of portfolio survivability and mortality experience.

5. *Liability-Relative Optimization:*
Asset allocation methodologies commonly ignore the funding risks, like inflation and currency, associated with an investor's goals. By incorporating the liability into the portfolio optimization process it is possible to build portfolios that can better hedge the risks faced by a retiree. While these "liability-driven" portfolios may appear to be less efficient asset allocations when viewed from an asset-only perspective, we find they are actually more efficient when it comes to achieving the sustainable retirement income.

From a more holistic perspective, each of these Gamma concepts can be thought of as actions and services provided by financial planners. This is a concept Bennyhoff and Kinniry (2011) called "Advisor's alpha" and Scott (2012) calls "household alpha." However, Bennyhoff and Kinniry do not attempt to quantify the potential benefit of these actions and discuss the implications in a more qualitative fashion and Scott focuses solely on the potential benefit from optimal Social Security claiming decisions. Scott does however note the potential use of a utility function to measure the tradeoffs involved in the Social Security decision. In this paper, we take a utility function approach to quantify the benefit of different income-maximizing decisions The goal of this paper is to provide some perspective, as well as quantify, the potential benefits that can be realized by an investor (in particular a retiree) from using a Gamma-optimized portfolio.

Measuring Gamma
One approach to quantifying the economic gain from making more intelligent financial planning decisions is to calculate the net present value of the additional income generated by the improved strategy. Scott (2012) uses this approach to quantify the economic benefit that American investors can

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obtain from strategically timing the start of Social Security benefits (e.g., delaying the initial claiming age). A benefit of using an objective measure like this is that it does not require any explicit assumptions about subjective investor preferences.

However, Scott notes that "one could argue for a utility measure." A utility approach is especially apt to evaluate strategies with uncertain results. For example, Scott and Watson (2013) use a utility maximization model to benchmark the efficacy of heuristic retirement income strategies with uncertain outcomes. Given the uncertain future cash flows associated with retirement income strategies, we developed a utility framework to measure Gamma, even though it requires that we make explicit assumptions about investors' attitudes towards the timing and risk of income. While objective approaches may exist in certain cases we needed an approach that is applicable to all strategies. To control for the subjective elements of our approach, we experimented with alternative value of the preference parameters. (See Appendix D.)

In its most general form, utility theory postulates that an investor ranks alternative combinations of levels of goods and services by mapping each combination into a single number, the level of utility. In situations that involve uncertainty, the alternative combinations are the possible outcomes of a random process. If each possible outcome is itself a single variable such as a level of wealth or income, and if the agent assigns a probability to each possible outcome, we can think of the agent assigning levels of utility to alternative probability distributions of the variable. With some additional assumptions, the level of utility of can be calculated as the mathematical expectation of a function of the variable. This function is called the expected utility function and the approach of modeling utility in this way is known as expected utility theory. Financial economists have successfully used expected utility for over 60 years to model rational investment decision making. In fact, Markowitz's (1959, 1987) famous mean-variance model of portfolio construction which led to the development of alpha and beta, has motivations from expected utility theory. In the context of measuring Gamma, the uncertainty is not merely regarding the level of income for one period, but rather with levels of incomes over multiple periods. While deterministic income time series can be ranked using an intertemporal utility function, ranking stochastic time series requires additional assumptions and modeling techniques. We present the details of our method for calculating the utility of simulated distributions of income time series in Appendix A. Here we summarize the procedure.

In each case that we test, for both the cases that serve as benchmarks and those that deploy one or more of the five strategies we describe above, we generate a matrix of income levels through Monte Carlo simulation. Each row of the matrix is made up of income levels at a given time from different simulations and each column is a Monte Carlo trial. For each column, we calculate the utility of the time series it contains using an intertemporal utility function. We then calculate what constant level of income would yield the same level of utility as that of the time series contained in the column. Making this calculation for each column reduces each Monte Carlo trial to a single number which can be interpreted as a level of income. Treating each trial as equally likely, we now have a probability distribution for this univarite income measure. We then use an expected utility function to calculate the utility of this probability distribution. Finally, we find what certain level of income would result in the same level of utility. This is the certainty-equivalent income of the case in question. Gamma is the percentage difference between the certainty-equivalent income of the case in question and that of the benchmark case.

[1] Sam Savage recalls Markowitz telling him that "he [Markowitz] had been indoctrinated at point-blank range in expected utility theory by my dad [Leonard J.

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While in principle utility functions can take any number of forms, to keep the calculations simple and to make it easy to vary investor preferences, we follow the common practice of using parametric utility functions. We do so for both the intertemporal and the expected utility functions. In Appendix A we present these functions along with our default set of preference parameters which we vary in Appendix D.

In calculating intertemporal utility, we take the uncertainty of lifespans into account with a parametric mortality model which we describe in Appendix B. Since the value of Gamma varies from one Monte Carlo simulation to the next, we use a bootstrap technique to estimate distributions for Gamma estimates. The standard deviations of these distributions provide estimates of the standard error of our Gamma estimates.

Gamma Tests

In order to determine the impact on Gamma from the five different strategies considered, we perform two entirely different "tests." We use the first test to determine the impact of Asset Location and Withdrawal Sourcing (i.e., tax efficient decisions) and the second to calculate contribution to Gamma from total wealth asset allocation, annuity allocation, dynamic withdrawal strategy, and liability-relative optimization.

While ideally a single generator would have been used to quantify the unique contribution of each of the five decisions, the tax calculations are relatively complex and the separation was done out of necessity. Therefore, in order to determine the aggregate Gamma from the two different tests, the results for the tests must be combined. We leave the functional form for aggregating Gamma from different tests for future research. Here, for simplicity purposes, we assume the improved certainty-equivalent income that could be generated are additive across the two tests. Since the tests are relatively independent (i.e., each quantifying some different aspect of potential "financial planning alpha"), we see no reason why this would not be the case.

Simulation and Bootstrapping

To simulate returns, we begin with a pool of 50,000 vectors of asset class returns generated using the Truncated Lévy Flight (TLF) distribution presented by Xiong and Idzorek (2011) with the asset class assumptions presented in Appendix B. (The TLF distribution is a skewed fat-tailed distribution that reflects the statistical properties that are found in historical asset class return data as documented by Kaplan [2012, chapters 18 and 19] and Xiong [2010].) To expedite calculations, we draw10,000 return-vectors with replacement before computing Gamma. We then draw with replacement from these 10,000 return-vectors, 5,000 50-year strings of return vectors to calculate Gamma. The returns selected and their sequence is determined by a seed, or initial starting variable. We bootstrap by varying the seed.We perform a bootstrap analysis by repeating each 5,000-trial Monte Carlo simulation multiple times, each time using a different seed. We use these results to estimate a distribution for Gamma in each case. We report the average of each distribution as our estimate of Gamma and the standard deviation as its standard error.

Test 1: Tax Efficiency

For the test on tax efficiency we created a simulator that contained two account types: a 401(k) account

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and a taxable account. For the 401(k) account, gains in the account are not realized until income is withdrawn from the account. All 401(k) income is assumed to be taxed at a 30% tax rate (which is lower than the highest current marginal tax rate of 39.6%). For the taxable account, tax is due for all gains that are realized. We assume that all bond returns are realized annually and taxed at the 30% tax rate. We assume that stock returns come from 50% long-term capital gains (or qualified dividends), which are taxed at 15%, and from 50% short-term capital gains (or non-qualified dividends), which are taxed at 30%. Within the taxable account, 40% of all gains during the year are assumed to be realized by the investor, which is a relatively tax-efficient portfolio. We assume that all income withdrawals from the taxable account are sourced from "basis" first if the annual gains are not enough to support the distribution. We assume that the beginning basis is 100% of the taxable account value upon retirement.

We assume that the account balances in the 401(k) account and the taxable account are equal. The key difference in this analysis is the location of the stocks and bonds and the sequence of the withdrawals from the two accounts. We assume that the equity allocation of the portfolio is a constant 40%, and this 40% equity allocation is maintained over the life of the portfolio. Stocks and bonds are first purchased in the 401(k) account in order to achieve the 40% equity target and then purchased in the taxable account, if necessary. In some cases this means realizing gains in order to maintain the target equity allocation. The key assumption, therefore, is that maintaining the target equity allocation is more important than tax efficiency. Also, if a consistent equity allocation were not maintained, the risk and return attributes of the portfolio could change considerably over the life of a given simulation, which would materially affect the results of the simulation. We assume a withdrawal rate of 4% of total financial wealth at retirement, increased annually by inflation during retirement.

We consider a total of nine different scenarios, three different asset location scenarios and three different withdrawal sequencing scenarios. Among the possible outcomes are an efficient scenario, a "split" scenario, and an "inefficient" scenario. The efficient scenario represents the most efficient possible solution, which is both allocating as much bonds as possible in the 401(k) account for the asset location test and withdrawing from the taxable account first for the sequencing test. The "split" scenario assumes everything is divided evenly among the options. The inefficient scenario represents the least-efficient possible solution, which is both holding as much stocks as possible in the 401(k) account for the asset location test and withdrawing from the 401(k) account first for the sequencing test. The scenario where both options are "split" (i.e., the double-split scenario) is assumed to be the "base scenario" and subsequent results from the other eight scenarios are compared against the results of the double-split scenario. Table 1 presents the results.

Table 1: Asset Location and Withdrawal Sequencing (Income Order) Results

Gamma of Strategy

Income Order	**Asset Location Portfolio Efficiency**		
	Efficient	1/n	Inefficient
401k First	-1.02%	1.92%	3.18%
Split	1.53%	4.05%	6.51%
Taxable First	3.23%	4.06%	10.56%

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As Table 1 shows, there are definite costs associated with inefficient investing during retirement. This cost can be attributed to the "return drag" associated with paying taxes versus delaying payment. The difference in retirement utility-adjusted income for the least efficient of the nine scenarios (inefficient asset location and 401(k) withdrawals first) to the most efficient (efficient asset location and taxable withdrawals first) is 3.23% with a standard error of 0.05% based on 100 simulations. This is a relatively large difference, but it is important to point out that this is a comparison of the worst possible outcome to the best. The double-split scenario is likely a better proxy, because as opposed to assuming the investor is being actively unintelligent (i.e., investing in an inefficient portfolio), we assume the investor is unsure what to do and therefore spreads the portfolio and income across the available options.

The very small standard error shows that the benefit of the more tax-efficient portfolio with a given asset allocation is virtually a constant. To see the impact of asset allocation on the benefit of tax efficiency, we estimated Gamma for asset mixes that vary in equity allocation from 10% to 60%. Table 2 presents the results. These results show that the greater the fixed income allocation (lower equity allocation), the greater the benefit of tax efficiency. This makes sense since fixed income returns are generally taxed at a higher rate than equity returns and the "tax alpha" impacts outcomes more for lower return portfolios.

Table 2: Impact of Asset Allocation on the Benefit of Tax-Efficiency

	Gamma Estimate	Standard Error
10%	4.41%	0.05%
20%	4.16%	0.04%
30%	3.70%	0.05%
40%	3.23%	0.05%
50%	2.67%	0.05%
60%	2.33%	0.05%

While this analysis included two common tax account types: 401(k) (or a Traditional IRA) and a taxable account, it did not include a Roth IRA account. A Roth account is excluded since most investors are not likely to have significant assets in this account type today; however, given the increasing flexibility of Traditional to Roth IRA rollovers, Roth IRAs are likely to become increasingly common account types for retirees. Roth IRA accounts are perhaps the most efficient account type for retirement income because there are no minimum required distributions, Roth IRA income does not affect Social Security benefit taxation, and Roth IRAs are very efficient from an estate tax planning perspective. Therefore, additional potential Gamma gains are likely possible for a retiree who has money in Roth IRA-type account.

Test 2: Total Wealth Asset Allocation, Annuity Allocation, Dynamic Withdrawal Strategy, and Liability-Relative Optimization

In order to determine the potential benefit associated with the four remaining Gamma factors, we created a base case similar to the one that we used for the tax-efficiency test but without a tax-deferred account. The overall "intelligence" of the base scenario will obviously affect the potential gains

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available through more advanced approaches. We assume a relatively intelligent base scenario, where the retirees (a male and a female both age 65) would follow the "4% rule." The base equity allocation is assumed to be 20% , which is the approximate average for heads of household (investors) from age 65 to 95 with a least $10,000 in financial assets based on the 2010 Survey of Consumer Finances.

For the total wealth asset allocation test, we assume a naïve portfolio allocation where the fixed income portion is invested in 20% Cash and 80% US Bonds. The equity portion is invested in 100% US Large-Cap Stocks. We place a boundary on the maximum and minimum potential equity allocation for the investment portfolio portion of the allocation. The boundaries are based on the equity allocations of the Morningstar target-date indices (Aggressive and Conservative, respectively). The range is between 33% and 67% equity for 65 year olds, decreases to between 26% and 53% for 75 year olds, and to between 24% to 48% by age 85 (where it roughly settles) .

With the second test we are able to estimate the Gamma of total wealth asset allocation, annuity allocation, a dynamic withdrawal strategy, and liability-relative optimization. For the total wealth asset allocation test we assume the overall optimal portfolio has an equity allocation of 45%, which is based on both public securities as well as non-publically traded instruments. We also assume that the mortality-weighted net present value of the annuity and/or Social Security income is "bond-like," i.e., with little or no risk of default. Given this assumed allocation, the remaining financial assets are invested in order to achieve a target equity allocation of 45%, (the assumed equity allocation of the overall optimal portfolio.) Note, though, the equity allocation is bounded between the high and low glide paths of the Morningstar target-date indices, as noted previously.

For the annuity allocation simulation, we assume that 25% of the total retirement assets are used to purchase a fixed immediate annuity that we assume has a payout rate of 5.68%. We obtained this rate from immediateannuities.com for a joint couple, male and female, both age 65, with 100% survivor benefit in December 2012.

The dynamic withdrawal strategy is based on the "Mortality Updating Failure Percentage" approach of Blanchett, Kowara, and Chen (2012) where the probability of outliving the distribution period parameter is 25% (which implies that the probability of success parameter is 75%). Under this approach, the percentage withdrawn from the portfolio will vary in a given year based on the assumed remaining expected mortality (i.e., expected retirement period) of the retiree/s and the amount that can be withdrawn that year that results in achieving the target probability of success. Table 3 shows a sample of withdrawal rates for different equity allocations over different time periods. For example, if the portfolio value is $100,000, the equity allocation is 40% and the remaining expected life expectancy is 20 years, the withdrawal amount for that would be $5,900 (which is 5.9% of the $100,000 portfolio).

[2] We provide a sensitivity analysis in Table D1 of Appendix D for the reader who is curious about the impact of different assumed basecase equity allocations. The impact is relatively minor.

[3] Contact the authors for more precise information about the equity glidepath range at each age.

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Table 3: Dynamic Withdrawal Strategy Portfolio Withdrawal Percentages by Equity Allocation and Number of Years Remaining

Years Remaining	Equity Allocation				
	20%	30%	40%	50%	60%
5	20.0%	19.9%	19.9%	19.8%	19.9%
10	10.4%	10.4%	10.5%	10.5%	10.5%
15	7.2%	7.3%	7.4%	7.4%	7.5%
20	5.7%	5.8%	5.9%	5.9%	6.0%
25	4.8%	4.9%	5.0%	5.0%	5.2%
30	4.2%	4.4%	4.5%	4.5%	4.7%
35	3.8%	3.9%	4.1%	4.1%	4.3%
40	3.5%	3.6%	3.8%	3.8%	4.0%

For the liability-relative optimization — an asset allocation that minimizes the variance of net assets — we create a reasonably efficient portfolio when thinking about the liability of a retiree: which is to create income for life, increased by inflation. Asset-centric approaches to portfolio optimization tend to ignore the risk structure of the underlying goal (which is retirement income in this case) and therefore can be suboptimal. The liability-relative optimal portfolio has a fixed income portion that is invested in 10% Cash, 20% US Bonds, 60% TIPS, and 10% Non-US Bonds and the equity portion is invested in 50% US Large stocks, 20% US Small stocks, 20% Non-US Large stocks, and 10% Emerging Markets.Social Security income is assumed to be half of the total annual real income target of the joint couple and therefore represents an asset that is 50% of the total value of the assets held by the retiree. While the precise Required Minimum Distributions (RMD) rules are not considered within the withdrawal process, the annual distributions for the dynamic approach do approximately equal RMDs since the withdrawals are based on remaining mortality.

Results

Since it is not possible to test for the impact of total wealth asset allocation, annuity allocation, dynamic withdrawal strategy, and liability-relative optimization individually, we determine the relative impact of each by changing an assumption within the test generator. Given the fact that there are four variables with two possible usage types ("yes" or "no") there are 16 different scenarios to consider to estimate Gamma, as shown in Table 4.

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Table 4: Test 2 Scenarios

Test Scenario	Tot Wealth Asset Allocation	Annuity Allocation	Dynamic Withdrawal Strategy	Liability Relative Optimization
1	No	No	No	No
2	Yes	No	No	No
3	No	Yes	No	No
4	Yes	Yes	No	No
5	No	No	Yes	No
6	Yes	No	Yes	No
7	No	Yes	Yes	No
8	Yes	Yes	Yes	No
9	No	No	No	Yes
10	Yes	No	No	Yes
11	No	Yes	No	Yes
12	Yes	Yes	No	Yes
13	No	No	Yes	Yes
14	Yes	No	Yes	Yes
15	No	Yes	Yes	Yes
16	Yes	Yes	Yes	Yes

The aggregate increase in certainty-equivalent income from the simplest scenario (1) to the most advanced scenario (16) is the Gamma for the combined effect of these Gamma factors. Our estimate of this composite Gamma is 19.40% with a standard error of 1.03% based on 250 simulations for these four factors.

To decompose this composite Gamma into the contribution of each of the four factors, we also calculate the percentage difference in certainty-equivalent income for the 32 pairs of scenarios listed in Table 5 as well as the pair (16,1) that we use to estimate the composite Gamma. Each pair consists of a scenario that includes the factor in the column heading and corresponding scenario that excludes it. We average the results of the eight pairs for each of the four factors. Let ΓA denote the percentage difference in certainty-equivalent income for the pair (16,1) and let $\bar{\Gamma}\_i$ denote the average of the percentage increase in certainty-equivalent income from including factor i in each of its eight pairs. We decompose ΓA into four components that represent the contribution of factor i as follows:

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Table 5: Pairs of Scenarios to Decompose Gamma by Factor

Tot Wealth Asset Allocation	Annuity Allocation	Dynamic Withdrawal Strategy	Liability Relative Optimization
(2, 1)	(3, 1)	(5, 1)	(9, 1)
(4, 3)	(4, 2)	(6, 2)	(10, 2)
(6, 5)	(7, 5)	(7, 3)	(11, 3)
(8, 7)	(8, 6)	(8, 4)	(12, 4)
(10, 9)	(11, 9)	(13, 4)	(13, 5)
(12, 11)	(12, 10)	(14, 10)	(14, 6)
(14, 13)	(15, 13)	(15, 11)	(15, 7)
(16, 15)	(16, 14)	(16, 12)	(16, 8)

We present results of the of this attribution analysis in Table 6. Among the four types, a dynamic withdrawal strategy added the most Gamma, at 9.88%. We also include a sensitivity analysis in Appendix D to provide a better understanding of the sensitivity of Gamma using different assumptions and input parameters.

Table 6: Attribution Analysis of Gamma for Test 2

Gamma Factor	Gamma Estimate	Standard Error
Total Wealth Asset Allocation	6.43%	0.53%
Annuity Allocation	1.44%	0.13%
Dynamic Withdrawal Strategy	9.88%	0.61%
Liability Relative Optimization	1.65%	0.31%
Total	**19.40%**	**1.03%**

Putting it All Together

Up to this point we have conducted two different tests to determine the relative impact of five different kinds of Gamma: using total wealth to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products, tax-efficient allocation decisions, and liability-relative portfolio optimization. While there may be slight differences in some of the assumptions used within the tests, the results of each of the tests should add value independently of the other four (as was demonstrated in the second test). If we add the results from the five different types of Gamma tested, we find a Gamma of 22.63%, i.e., $122.63 for every $100 generated by the base set of assumptions. We display this concept visually in Figure 1.

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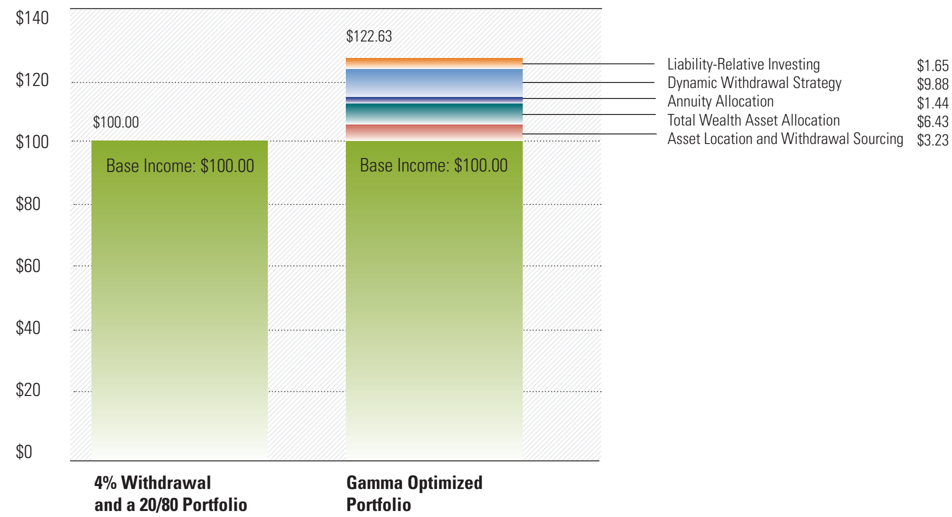
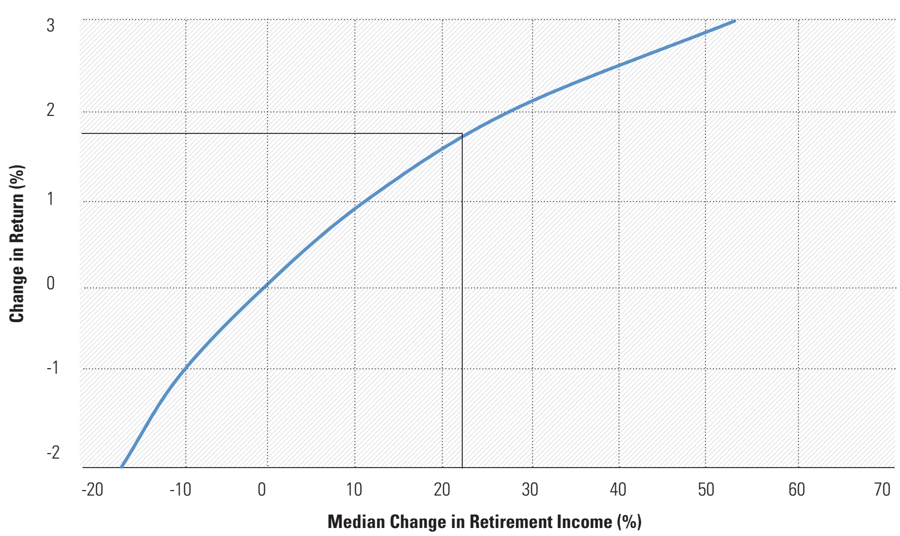
Figure 1: Relative and Combined Impact of the Strategies Considered



An increase in certainty-equivalent utility-adjusted income of 22.6% represents an impressive improvement in potential retirement income, but how does it relate from a traditional alpha perspective? In order to determine how much additional annual return, or alpha, is equivalent to the 22.6% Gamma, we conduct an additional analysis. We determine the total income generated for a 4% initial withdrawal rate and compare it to the income generated by portfolios with returns that are either higher or lower than the base portfolio(-2%, -1%, 0% (no change), +1%, +2%, and +3%). We compare the difference in the amount of income generated by the 4% withdrawal portfolio against the income generated by the 0% change (i.e., no change) portfolio. We show these results in Figure 2.

Figure 2: Relationship between Additional Retirement Income and Changes in Returns for a 4% Initial Withdrawal Rate



[4] For the tax-efficiency Gamma, we use the result for the 40% equity allocation, 3.23%, to split the difference between the equity allocations for the target total wealth (60%) and the naïve portfolio (20%).

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By fitting a third-order polynomial to the curve depicting the 4% Initial Withdrawal, we estimate the equivalent return impact of a +22.6% increase in retirement income to be 1.59%. Table 7 shows how we attribute this Gamma-equivalent alpha among the five Gamma factors. This is likely to be significantly higher than any type of portfolio "alpha" that a financial advisor would be able to generate through fund selection or market timing. Also, while traditional portfolio alpha is a negative-sum game (since everyone cannot, on average, outperform the market), our results show that Gamma is not a zero-sum game and can be achieved by any investor who takes a smarter approach to generating retirement income.

Table 7: Additional Certainty-Equivalent Income Amounts and Gamma-Equivalent Alpha Values

Gamma Type	Additional Income Generated	Gamma Equivalent Alpha
Total Wealth Asset Allocation	6.43%	0.45%
Annuity Allocation	1.44%	0.10%
Dynamic Withdrawal Strategy	9.88%	0.70%
Liability Relative Optimization	1.65%	0.12%
Asset Location and Withdrawal Sourcing	3.23%	0.23%
Total	**22.63%**	**1.59%**

Implementation

In this paper we explored five fundamental financial planning decisions/techniques: a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products (i.e., annuities), tax-efficient decisions, and liability-relative asset allocation optimization. Before concluding, let us revisit each of these decisions within the context of what it means to be following a Gamma-optimized approach:

- Total wealth framework: asset allocation decisions should be made considering an investor's total wealth, not just financial assets. Income sources such as pensions or Social Security and other forms of human capital, should be considered when building a portfolio, since each comprises an individual's holistic wealth and has varying risk characteristics.

- Dynamic withdrawal strategy: portfolio withdrawal decisions should be revisited on some regular basis, ideally at least annually, to ensure the portfolio withdrawal amount is still prudent and reasonable given return expectations and expected remaining length of the retirement period.

- Incorporating guaranteed income products: annuities provide a guarantee that cannot be created from a traditional portfolio: income for life. Annuities are a valuable form of insurance, and should at least be considered for each retiree.

- Tax-efficient decisions: taxes are a known drag on performance and the actual return realized by an investor. Therefore, it is very important to consider taxes when designing a portfolio for a client and withdrawing income during retirement.

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- Liability-relative asset allocation: people tend to save or accumulate wealth in order to fund some kind of goal (or liability). Therefore, it is important to consider the risk attributes of that liability when building the portfolio. Traditional mean-variance optimization focuses entirely on the risk of the assets, and ignores the risks of the goals that the assets are meant to fund.

- Finally, investment advisors will need to bear in mind that the strategies that follow from a Gamma analysis are broad guides and not precise recommendations.

Here we have explored only five of the many different financial planning decisions that should be considered. There are definitely other decisions that can be as if not more important than the five reviewed here, especially for investors across different stages of their lifecycle (e.g., the savings decision for someone in the accumulation stage of retirement finance).

Conclusion

In this paper, we present a concept that we call "Gamma." We define Gamma as the additional value achieved by an individual investor from making more intelligent financial planning decisions, measured by the percentage increase in certainty-equivalent retirement income over a base case. While Gamma varies for different types of investors, in this paper we focus on five types of Gamma relevant to retirees: using a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products, tax-efficient allocation decisions, and liability-relative portfolio optimization. Among the five types of Gamma tested, using a dynamic withdrawal strategy was determined to be the most important, followed by total wealth asset allocation approach, and then making tax-efficient allocation decisions.

In the aggregate we estimate a retiree can be expected to generate 22.6% more certainty-equivalent income utilizing a Gamma-efficient retirement income strategy when compared to our base scenario of a 4% withdrawal rate and a 20% equity allocation portfolio. This has the same impact on expected utility as an annual return increase of +1.59% (i.e., Gamma-equivalent alpha), which represents a significant improvement in portfolio efficiency for a retiree. Unlike traditional alpha, which is a zero-sum game and likely a negative sum game after fees, we find that Gamma (and Gamma-equivalent alpha) can be achieved by anyone following an efficient financial planning strategy. Of course these figures are based on models and are subject to statistical error as we have indicated with estimated standard errors. But the results are strong enough to highlight the difference that intelligent financial planning can make for investors.

There are two caveats to our results. First, as we showed in our sensitivity analysis (Appendix D), at least at the extremes, investor attitudes toward the timing of income can be important in assessing the value of intelligent financial planning. Secondly, there may be any additional costs in implementing the Gamma-generating strategies tested that we have not accounted for. Further research is needed to address the cost side of the financial planning equation.

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Appendix A: Details on Measuring Gamma

For a single period, expected utility theory states that an investor ranks alternative uncertain amounts of income by the expected utility of each. Letting $\tilde{I}$ denote the random amount of income in the given period, the expected utility of $\tilde{I}$ is

$$EU[\tilde{I}] = E[u(\tilde{I})] \tag{A1}$$

Where $u(.)$ is an increasing concave utility function that reflects the risk tolerance of the investor. Since values of $u(.)$ are abstract measures of "utility," a common practice is to convert expected utility to the utility-adjusted certainty-equivalent level of income,

$$CE[\tilde{I}] = u^{-1}(E[u(\tilde{I})]) \tag{A2}$$

This means that the investor is indifferent between the random amount of income $\tilde{I}$ and the certain amount of income $CE[\tilde{I}\,]$. Gamma measures how much additional utility-adjusted income a strategy in question adds over and above the utility-adjusted income from a set of base-case decisions.

There are several parametric forms of $u(.)$ common in the literature. The most common one is the Constant Relative Risk Aversion (CRRA) utility function, which we write as:

$$u(x) = \begin{cases} \frac{\theta}{\theta-1} x^{\frac{\theta-1}{\theta}}, & \theta > 0, \theta \neq 1 \\ ln(x), & \theta = 1 \end{cases} \tag{A3}$$

where θ is the risk tolerance parameter. Because of its analytical simplicity and its ability to represent a wide range of attitudes toward risk with a single parameter, the CRRA utility function is used not only in single-period models, but in multi-period models as well. Letting $\tilde{I}_t$ denote the random amount of income in period t, the expected utility of the sequence of incomes from periods 0 to T in these models is:

$$EU = \sum_{t=0}^{T} d_t \frac{\theta}{\theta-1} E\left[\tilde{I}_t^{\frac{\theta-1}{\theta}}\right] \tag{A4}$$

where d_t is the discount factor for period *t*. However, note that the parameter θ plays a role in the calculation of utility even if there is no uncertainty. This is because, in a multi-period context, θ plays two roles: (1) the investor's risk tolerance parameter and (2) the investor's elasticity of intertemporal substitution (EOIS) preference parameter.

Epstein and Zin (1989) point out that there is no reason in principle that the risk tolerance parameter and the EOIS parameter are equal. The only reason for setting them equal is mathematical expediency. Epstein and Zin formulate expected utility in a way that makes these distinct parameters by recursively nesting the certainty equivalence function inside of the intertemporal utility function:

$$V_t = \left[I_t^{\frac{\eta-1}{\eta}} + \frac{d_{t+1}}{d_t} CE_t[V_{t+1}]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{A5}$$

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where V_t is the utility of the stream of income beginning at time *t* (measured in the same unit as income), and η is the investor's elasticity of intertemporal substitution preference parameter[5]. The *t* subscript on the certainty equivalent operator denotes that it is conditional on what is known at time *t*.

Epstein and Zin formulated their utility function to generalize the recursive expected utility maximization problem formulated by Lucas (1978) whereas Gamma is derived from a measure of the utility of a given set of simulated income paths. So, we formulate a utility function with the same EOIS and risk parameters as the Epstein-Zin utility function that can be evaluated without recursion. We achieve this by reversing the order of the nesting of intertemporal and risk components of utility. Specifically, for each simulated income path, we calculate its utility-equivalent constant income level based on the EOIS parameter, which we denote as II. That is, for a given simulated income path, II is the constant amount of income with the same utility as the actual income path[6]. This is given by

$$II = \left(\frac{\sum_{t=0}^{T} q_t (1+\rho)^{-t} I_t^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{T} q_t (1+\rho)^{-t}} \right)^{\frac{\eta}{\eta-1}} \qquad \text{[A6]}$$

I_t = the level of income in year t
q_t = the probability of surviving to at least year t
$\boldsymbol{T}$ = the last year for which $q_t > 0$
ρ = the investor's subjective discount rate (so that d_t in equation [A5] is $q_t(1+\rho)^{-t}$)

Note that while equation [A6] contains two preference parameters (ρ and η) that describe how the investor feels about having income to consume at different points in time, it makes no reference to how the investor feels about risk. As we discuss above, we treat the elasticity of intertemporal substituation as a parameter distinct from the risk tolerance parameter. We introduce the risk tolerance parameter next by treating the entire path as unknown and evaluating expected utility.

We measure expected utility using the CRRA utility function with its risk tolerance parameter θ that we introduced in equation [A3]:

$$EU = \sum_{i=1}^{M} p_i \frac{\theta}{\theta-1} (II_i)^{\frac{\theta-1}{\theta}} \qquad \text{[A7]}$$

where *M* is the number of paths, the subscript *i* to denote which of *M* paths is being referred to, and p_i is the probability of path *i* occurring which we set to 1/*M*.

We define *Y* as the constant value for II that we yield this level of expected utility. This is the certainty-equivalent of the stochastic utility-adjusted income II. *Y* is given by

$$Y = \left[\sum_{i=1}^{M} p_i (II_i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \qquad \text{[A8]}$$

[5] The η parameter is the same as the EOIS parameter σ in Epstein-Zin (1989). However, in their equations, they use the symbol ρ where we use the expression $\frac{\eta}{\eta-1}$
[6] Williams and Finke (2011) use a similar concept to assess the relative attractiveness of different withdrawal rates.

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We can now formally define the Gamma of a given strategy or set of decisions as

$$Gamma(Strategy) = \frac{Y(Stategy) - Y(Benchmark)}{Y(Benchmark)} \quad \text{[A9]}$$

As a base case we use the following parameter values: $\rho = 2.5\%$, $\eta = 0.5$, and $\theta = 0.33$. In Appendix D, we perform sensitivity analysis to explore the impact of how Gamma is affected by the choice values for these parameter values.

Appendix B: The Mortality Model

We model mortality using the "Gompertz Law of Mortality;" named for Benjamin Gompertz. Gompertz discovered that a person's probability of dying increases at a relatively constant exponential rate as age increases. We use the same formulation of Gompertz law for mortality as Milevsky and Robinson (2000), where the probability of survival to age $t \leq 115$,[7] conditional on a life at age (*a*), is given by:

$$q_t = \exp\left\{\exp\left\{\frac{a-m}{b}\right\}\left(1 - \exp\left\{\frac{t-a}{b}\right\}\right)\right\} \quad \text{[B1]}$$

where m is the modal lifespan and b is the dispersion coefficient. We use Gompertz parameters that are fitted to the discrete "Annuity 2000 Basic Table" mortality table presented in Johansen (1998) using the procedure described below. The probability of at least one member of a heterosexual couple surviving to age *t* is

$$q_t = q_t^{Male} + q_t^{Female} - q_t^{Male} q_t^{Female} \quad \text{[B2]}$$

As Figure B1 shows, the results of the Gompertz model are very close to the results obtained directly from the mortality rates.

Fitting the Gompertz Model

Table 1, the "Annuity 2000 Basic Table" in Johansen (1998) contains mortality rates per 1,000 individuals for males and females ages 5–115. In equation 3, $\mathrm{Mort}_t^{\mathrm{Sex}}$ denotes the mortality rate for a person of the given sex and age t. We use these data to calculate survival rates for persons ages 65–115 for each sex as follows:

$$\bar{q}_t^{Sex} = \begin{cases} 1, & t = 65 \\ \bar{q}_{t-1}^{Sex}\left(1 - \frac{Mort_t^{Sex}}{1000}\right), & t > 65 \end{cases} \quad \text{[B3]}$$

From these we calculate the probability of a person of a given sex dying at age $t > 65$:

$$\bar{p}_t^{Sex} = \bar{q}_{t-1}^{Sex} - \bar{q}_t^{Sex} \quad \text{[B4]}$$

The age at which $\bar{p}_t^{Sex}$ reaches its maximum value is the modal age for the given sex (*m* in equation [9]). These turn out to be 86 for males and 90 for females.

5 Age 115 is the oldest age assumed for the analysis.

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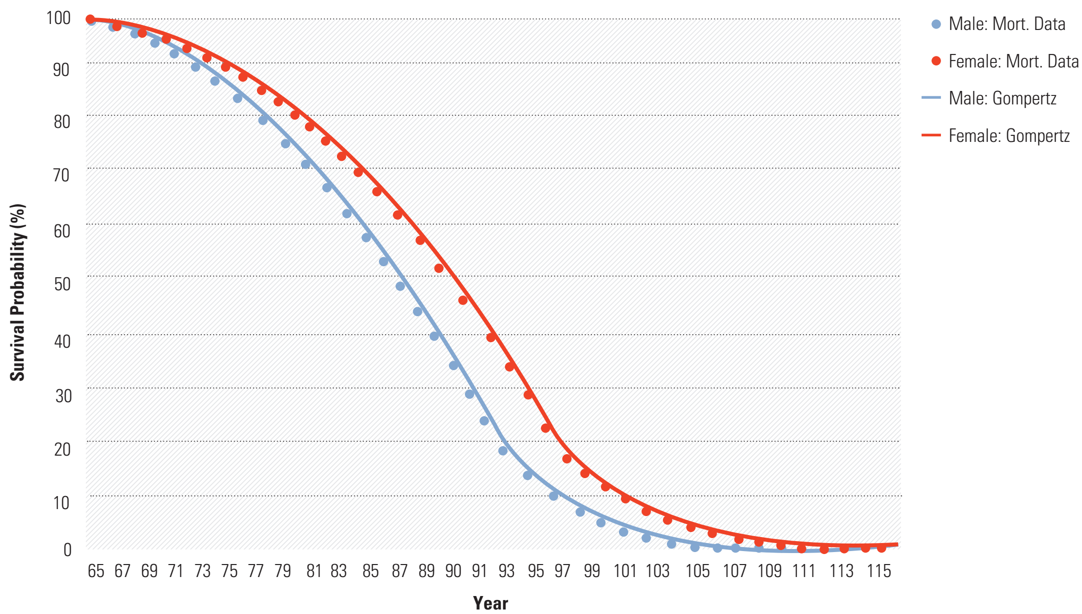
For each sex we estimate the dispersion coefficient (*b* in equation [B1]) by minimizing the sum of squared differences:

$$SSD = \sum_{t=65}^{115}(q_t - \bar{q}_t)^2 \quad \text{[B5]}$$

q_t being the survival probability given by the Gompertz model in equation [A6]. The results are 10.48 for males and 8.63 for females. As Figure B1 shows, the results of the Gompertz model are very close to the results obtained directed from the mortality rates.

Figure B1: Fitting the Gompertz Model to the Mortality Data



Appendix C: Nominal Market Assumptions

Asset Class	Index	Returns	Std Dev	Kurtosis	Skewness
Cash	IA SBBI US 30 Day TBill TR USD	1.92%	3.18%	0.46	0.45
US Bonds	BarCap US Agg Bond TR USD	4.05%	6.51%	2.60	1.14
Non-US Bonds	IA Global ex-US Bond Composite	4.06%	10.56%	0.08	0.69
US TIPSs	BarCap Gbl Infl Linked US TIPS TR USD	3.57%	7.03%	-0.28	0.45
US Large-Cap Stocks	IA SBBI S&P 500 TR USD	9.61%	19.50%	0.20	-0.70
US Small-Cap Stocks	Russell 2000 TR USD	11.77%	24.68%	-0.33	-0.26
Non US Large Cap Stocks	MSCI EAFE GR USD	10.29%	21.05%	0.42	0.06
Emerging Markets Stocks	IA Emerging Markets Composite	15.17%	31.52%	-0.70	0.11
Inflation	IA SBBI US Inflation	2.23%	3.13%	1.65	1.48

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Correlations	Cash	US Bonds	Non-US Bonds	US TIPSs	US Large	US Small	Non-US Stk	Emerg Mkts	Inflation
Cash	1.00	0.27	-0.13	-0.27	0.20	0.11	0.08	-0.20	0.66
US Bonds	0.27	1.00	0.18	0.51	0.28	0.09	0.01	-0.25	-0.21
Non-US Bonds	-0.13	0.18	1.00	0.35	0.06	-0.07	0.46	0.17	-0.08
US TIPSs	-0.27	0.51	0.35	1.00	-0.13	0.01	-0.06	0.07	0.52
US Lg-Cap Stocks	0.20	0.28	0.06	-0.13	1.00	0.66	0.60	0.38	0.09
US Sm-Cap Stocks	0.11	0.09	-0.07	0.01	0.66	1.00	0.42	0.42	0.26
Non-US Lg-Cap Stocks	0.08	0.01	0.46	-0.06	0.60	0.42	1.00	0.62	0.06
Emerging Mkt Stocks	-0.20	-0.25	0.17	0.07	0.38	0.42	0.62	1.00	0.10
Inflation	0.66	-0.21	-0.08	0.52	0.09	0.26	0.06	0.10	1.00

The above expected returns and standard deviations are based on Ibbotson's Capital Market Assumptions (CMAs) as of December 31, 2011. The correlations, skewness, and kurtosis values used to generate the multivariate non-normal distribution are based on annual calendar year returns for the respective asset classes from 1973 to 2011. Note, the respective correlations, skewness, and kurtosis values for US TIPS are only from 1998 to 2011 since US TIPS were not introduced until 1997. While synthetic proxies do exist for TIPS, we decided to solely use actual historical data due to the difficulties associated with accurately backfilling this complex asset class.

Appendix D: Sensitivity Analysis

To see the impact of the asset allocation of the base case, we recalculated Test 2 (bootstrapping by repeating each 5,000-trial Monte Carlo simulation 100 times) using different equity allocations. Table 7 shows the results.

Table D1: Impact of Base Case Asset Allocation on Test 2 Results

Equity Allocation	Gamma Estimate	Standard Error
10%	21.72%	1.09%
20%	19.27%	0.96%
30%	18.34%	0.90%
40%	18.21%	0.86%
50%	18.52%	0.82%
60%	19.14%	0.77%

[8] The reader may note the assumed level of annual inflation (2.23%) is higher than the assumed return on cash (1.92%). Therefore, the authors are forecasting a negative real (inflation-adjusted) return on cash for this paper. These forecasts are based on Ibbotson's Capital Market Assumptions as of March 30, 2012. While this assumption may seem questionable, it is certainly valid given the current cash returns of effectively 0%.

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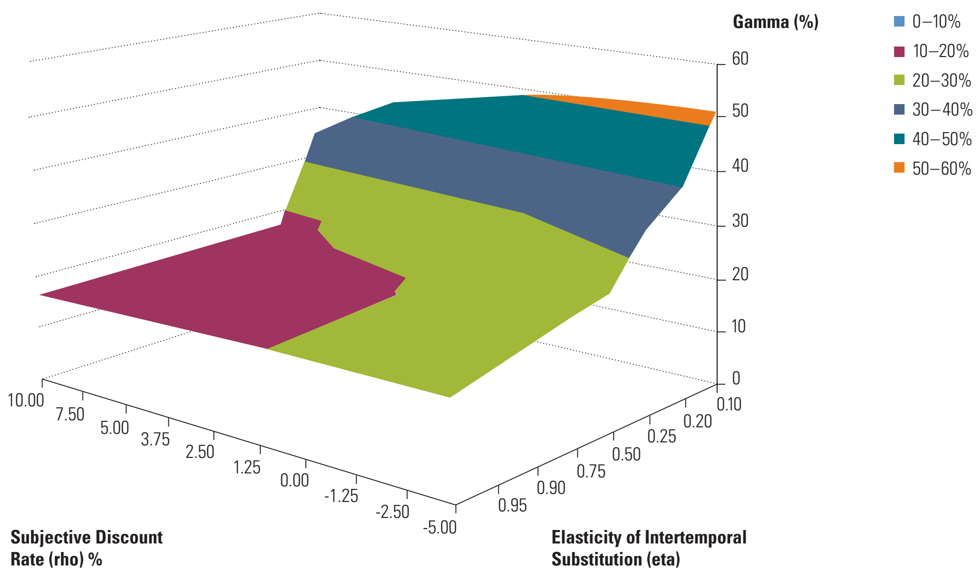
We note that for all equity allocations greater than our base case of 20%, the Gamma estimate is almost within one standard error of the base case result. Hence we conclude that Gamma varies little across base case asset allocations.

To see the impact on Gamma of varying the preference parameters for Test 2, we took the simulation from our initial bootstrap analysis that had the closest value to the average, which was 19.36%. Using this simulation, we varied the values of risk tolerance (θ), the subjective discount rate (ρ), and the elasticity of intertemporal substitution (η).

We found that varying θ had little impact on Gamma. Figure D2 shows the impact of varying ρ and η keeping θ fixed at 0.33. As this figure shows, there is a large impact on Gamma when η is set very low. In other words, the Gamma for investors who have a very low elasticity of intertemporal substitution is much higher than those for whom it is high.

Figure D2: Sensitivity Analysis on Gamma



The Elasticity of Intertemporal Substitution

To better understand the role of the elasticity of intertemporal substitution (EOIS) parameter, η and its impact on Gamma, consider a model in which a given amount of total wealth (financial assets plus human capital) is used to finance consumption over an infinite time horizon. Assume that the market offers a risk-free flat yield curve. If the single market interest rate equals the subjective discount factor (ρ), the optimal level of consumption is the same in every year regardless of the marginal rate of intertemporal substitution. However, if the market interest rate is less than the subjective discount rate, the optimal consumption path is downward sloping. As Figure 3 shows, if the EOIS is high (η=0.9), the optimal level of consumption starts high and the path is steeply sloped. However, if the EOIS is low (η=0.1), the optimal consumption path is nearly flat.

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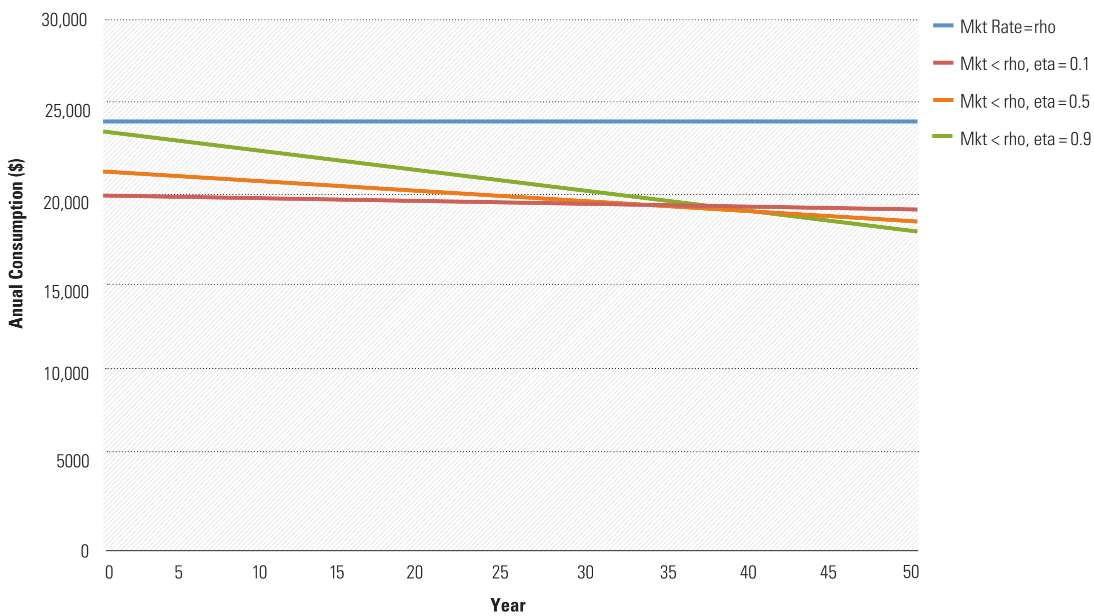
This model and our results on the sensitivity of Gamma to the η parameter illustrate that gaging investors' willingness to reschedule income can be an important input to financial planning. While most financial planning questionnaires are designed to illicit information about an investor's investment horizon and risk tolerance, we are not aware of any that seek to determine an investor's elasticity of intertemporal substitution. We hope that our research inspires incorporation of this aspect of investor preferences into financial planning practices.

Figure D3: Optimal Consumption Paths for Different Levels of the Elasticity of Intertemporal Substitution



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[3]The reader may note the assumed level of annual inflation (2.23%) is higher than the assumed return on cash (1.92%). Therefore, the authors are forecasting a negative real (inflation-adjusted) return on cash for this paper. These forecasts are based on Ibbotson’s Capital Market Assumptions as of March 30, 2012. While this assumption may seem questionable, it is certainly valid given the current cash returns of effectively 0%.

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